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EVOLUTION OF LINEAR WAVES IN A LIQUID IN THE PRESENCE OF A CURTAIN OF BUBBLES

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We investigate certain features of the evolution of waves in an acoustically compressible liquid in passage through a curtain of bubbles between two parallel planes. We consider the problem of reflection from a plane solid wall separated from the liquid by a curtain of bubbles. Investigations showed that in relation to the duration of the pulse it is possible to select a curtain with corresponding parameters to moderate the effect of the wave on the wall.

The introduction of a small amount of gas into a liquid in the form of gas bubbles distributed throughout the volume of the liquid substantially changes the acoustic properties of the liquid [1-4]. In particular, an anomalous decrease in the rate of propagation of sonic perturbations and substantial enhancement of the dissipation mechanisms occur. All this makes it possible to use bubble curtains to suppress pressure pulses in a liquid. In [4, 5] certain special features of the evolution of nonlinear waves in a liquid and a gas in passage through a curtain of bubbles was studied and the effect of a shock wave on a solid wall in the presence of a curtain of bubbles with a variable gas content was investigated. In the present work we investigate certain features of the evolution of waves in an acoustically compressible liquid in passage through a curtain of bubbles in the region between two parallel planes. Moreover, we consider the problem of reflection of waves from a plane solid wall separated from the liquid by a curtain of bubbles.

1. Basic Equations. Let the curtain be a mixture of a liquid with spherical gas bubbles of identical radius. Here the wavelength is much larger than the width of the curtain of bubbles. This makes it possible to assume that the distributions of the perturbations of the parameters (radius, volumetric content of bubbles, pressure in the liquid) are homogeneous over the thickness of the curtain. Let us consider a certain cylindrical volume of bubble liquid of unit cross section between planes that are the boundaries of the curtain. Then for the mass of the liquid and the number of bubbles in this volume in the absence of phase changes and fragmentation and flocculation of the bubbles we may write

$$\overset{0}{\rho}_{\text{liq}} (1 - \alpha_g)\, l = m = \text{const}\,, \quad nl = N = \text{const}\,, \quad \alpha_g = \frac{4}{3}\pi a^3 n\,. \tag{1}$$

From Eqs. (1) in a linearized approximation we may obtain

$$(1 - \alpha_g)\, \overset{0}{\rho}_{\text{liq}} - \frac{3\overset{0}{\rho}_{\text{liq0}}\, \alpha_{g0}}{a_0}\, a + \frac{\overset{0}{\rho}_{\text{liq0}}}{l_0}\, l = 0\,. \tag{2}$$

To describe radial motions of the bubbles in the curtain we take the Rayleigh–Lamb equation, which in the case of polytropic behavior of the gas in the bubbles in a linearized approximation is presented in the form

$$\frac{d^2 a}{dt^2} + 2\beta \frac{da}{dt} + \omega_R^2 a = \frac{-\, p_{\text{liq}}}{\overset{0}{\rho}_{\text{liq0}} a_0}\,, \tag{3}$$

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$$\beta = \frac{2\nu}{a_0^2}, \quad \omega_R = \sqrt{\left(\frac{3\gamma p_0}{\overset{0}{\rho}_{\text{liq}0}}\right)}\, a_0^{-1}$$

The equation of state of the liquid will be taken in the acoustic approximation:

$$p_{\text{liq}} = C_{\text{liq}}^2 \rho_{\text{liq}}\,. \tag{4}$$

Let us consider the evolution of a plane one-dimensional wave in the liquid interacting with the curtain of bubbles. By virtue of the assumption that the wavelength is much larger than the bubble-curtain thickness, in what follows we will take the curtain as a certain reflecting surface that intersects the axis Ox at $x = 0$. Outside this surface ($|x| > 0$) the following equations of continuity and pulses are valid for perturbations:

$$\frac{\partial \overset{0}{\rho}_{\text{liq}}}{\partial t} + \overset{0}{\rho}_{\text{liq}0}\frac{\partial v}{\partial x} = 0\,, \quad \overset{0}{\rho}_{\text{liq}0}\frac{\partial v}{\partial t} = -\frac{\partial p_{\text{liq}}}{\partial x}\,, \tag{5}$$

We will assume that the initial perturbation is a wave that propagates in the part to the left of the reflecting surface ($x < 0$). When the wave interacts with the bubble curtain, we assume equality of pressures over its two boundaries. Then this condition of continuity of pressure on the reflecting surface can be written in the form

$$p^{(0)} + p^{(r)} = p^{(t)} = p_{\text{liq}}\,, \quad x = 0\,. \tag{6}$$

We will assume that the velocity of the liquid suffers a discontinuity on the reflecting surface, and its magnitude is equal to the rate of change of the bubble-curtain thickness:

$$v^{(t)} - (v^{(0)} - v^{(r)}) = \frac{dl}{dt}\,, \quad x = 0\,. \tag{7}$$

In the case where a perfectly rigid wall is located behind the curtain, we assume that $\vartheta^{(t)} = 0$, with $p^{(t)}$ expressing the pressure experienced by the wall.

2. Dispersion Analysis. Suppose a plane harmonic wave is incident on a reflecting surface. Then the motion in the part to the left of the reflecting surface ($x < 0$) is a superposition of two waves: an incident one

$$p^{(0)} = A_p^{(0)} \exp i\,(kx - \omega t)\,, \quad v^{(0)} = A_v^{(0)} \exp i\,(kx - \omega t) \tag{8}$$

and a reflected one

$$p^{(r)} = A_p^{(r)} \exp i\,(-kx - \omega t)\,, \quad v^{(r)} = A_v^{(r)} \exp i\,(-kx - \omega t)\,, \tag{9}$$

and the part to the right of the reflecting surface ($x > 0$) there is only one, transmitted wave:

$$p^{(t)} = A_p^{(t)} \exp i\,(kx - \omega t)\,, \quad v^{(t)} = A_v^{(t)} \exp i\,(kx - \omega t)\,, \tag{10}$$

where $k = \omega / C_{\text{liq}}$ is the wave number. Here, on the basis of Eqs. (5) the amplitudes of the pressures and the velocity are interrelated as

$$A_p^{(0)} = \overset{0}{\rho}_{\text{liq}0}\, C_{\text{liq}}\, A_v^{(0)}\,, \quad A_p^{(r)} = -\overset{0}{\rho}_{\text{liq}0}\, C_{\text{liq}}\, A_v^{(r)}\,, \quad A_p^{(t)} = \overset{0}{\rho}_{\text{liq}0}\, C_{\text{liq}}\, A_v^{(t)}\,. \tag{11}$$

The interrelation between the three (incident, reflected, and transmitted) waves is determined by boundary conditions (6) and (7) on the reflecting surface.

In the case of harmonic waves the perturbations of the parameters in the curtain are represented in the form

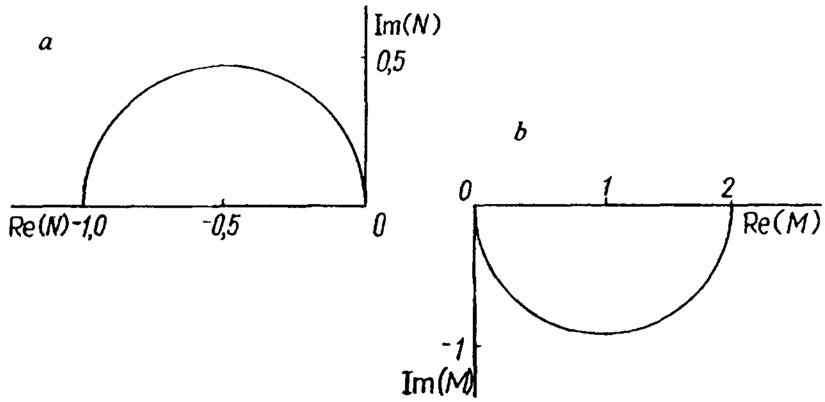


Fig. 1. Hodographs of the coefficients of reflection (a) and transmission (b) in the complex plane.

$$a = A_a \exp(-i\omega t), \quad p = A_p \exp(-i\omega t), \quad l = A_l \exp(-i\omega t), \tag{12}$$

We introduce the coefficients of reflection N and transmission M of linear waves by the following expressions:

$$N = \frac{A_p^{(r)}}{A_p^{(0)}}, \quad M = \frac{A_p^{(t)}}{A_p^{(0)}}. \tag{13}$$

After corresponding transformations and calculations, from the presented equations (2) and (4) with account for Eqs. (8)-(13) we obtain for the coefficients of reflection and transmission

$$N = \left(\frac{2C_p^2}{i\omega l_0 C_{\text{liq}}} - 1\right)^{-1}, \quad M = 1 + N, \tag{14}$$

$$\frac{1}{C_p^2} = \frac{1 - \alpha_{g0}}{C_{\text{liq}}^2} + \frac{1}{C^2 (1 - \omega^2/\omega_R^2 - i\omega/\omega_\nu)}, \quad C = \sqrt{\left(\frac{3p_0}{\overset{0}{\rho}_{\text{liq}0}\, \alpha_{g0}}\right)},$$

$$\omega_R = \sqrt{\left(\frac{3\gamma p_0}{\overset{0}{\rho}_{\text{liq}0}}\right)}\, a_0^{-1}, \quad \omega_\nu = \frac{3\gamma p_0}{4\nu \overset{0}{\rho}_{\text{liq}0}}.$$

In the case where it is possible to neglect effects associated with radial inertia of the liquid in the curtain ($\omega << \omega_M$) and viscosity ($\omega << \omega_\nu$), for the parameter C_p, which determines the complex compressibility of the bubble curtain, we may write

$$C_p = C_e, \quad \frac{1}{C_e^2} = \frac{1 - \alpha_{g0}}{C_{\text{liq}}^2} + \frac{1}{C^2} \tag{15}$$

We note that the expression obtained for C_p coincides with the formula for the equilibrium speed of sound in a liquid with bubbles [4]. In the majority of cases the conditions $\omega << \omega_M$, ω_ν follow automatically from the main assumption of the work that here long waves (substantially exceeding the thickness of the curtain) are considered. Then, expression (14) can be presented in the form

$$N = \frac{\exp(i\varphi) - 1}{2}, \quad \tan\varphi = \frac{2S}{S^2 - 1}, \quad S = \frac{\omega_*}{\omega}, \quad \omega_* = \frac{C_e^2}{C l_0}. \tag{16}$$

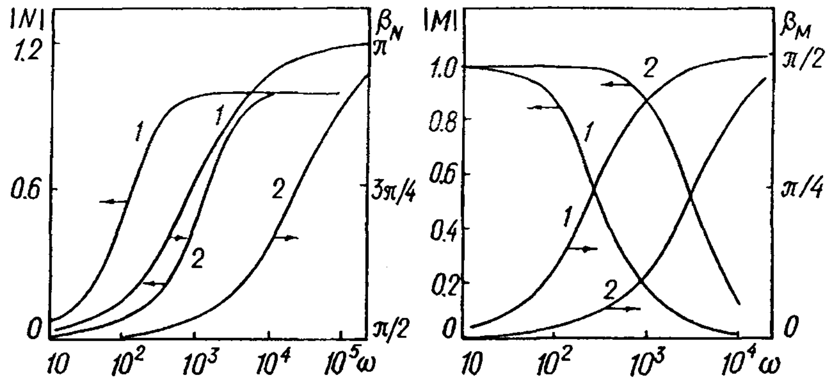


Fig. 2. Dependences of the modulus and argument of the coefficients of reflection and transmission in the liquid ($|N|$, $\beta_N = \arg N$; $|M|$, $\beta_M = \arg M$) on the frequency ω ($\sec^{-1}$) for different volumetric contents of gas in the curtain.

From Eq. (16) it follows that with a change in the frequency from 0 to $+\infty$ in the complex plane the region of the values of the coefficient of reflection N is a semicircle in the upper half-plane of radius (1/2) with center on the real axis at the point $-1/2$ (Fig. 1a). Moreover, for low-frequency perturbations that satisfy the condition $\omega \ll \omega_*$, $\omega_* = C_e^2/C_{\text{liq}}l_0$, we have $N \approx 0$, $M \approx 1$ and the curtain does not exert a noticeable effect on the dynamics of the wave. For high-frequency perturbations ($\omega \gg \omega_*$) we have $N \approx -1$, and therefore the reflection from the curtain of bubbles is similar to the reflection from a free surface. Moreover, in this case ($\omega \gg \omega_*$) the curtain becomes completely opaque for transmitted waves ($M \approx 0$).

Figure 2 presents dependences of the modulus and argument of the reflection and transmission coefficients on frequency that were calculated from expression (14) (curves 1 and 2 for $l_0 = 0.1$ and 0.01 m, respectively, $\alpha_{g0} = 10^{-2}$). Here and below, all calculations were carried out for a water-air mixture at $p_0 = 0.1$ MPa, $T_0 = 300$ K, $\rho^0_{\text{liq}0} = 10^3$ kg/m^3, $a_0 = 10^{-3}$ m, $C_{\text{liq}} = 1500$ m/sec, $\gamma = 1.4$, $\nu = 2 \cdot 10^{-6}$ m^2/sec. It is seen that the modulus of the reflection coefficient and its argument increase with increase in the curtain thickness. Whereas at the frequency, say, $\omega = 10^3$ sec^{-1}, when the curtain is thin ($l_0 = 10^{-2}$ m), we have $|N| \approx 0$, $|M| \approx 1$, $\beta_N \approx \pi/2$ and therefore a harmonic wave passes through the curtain nearly completely, with an increase in the curtain thickness to $l_0 = 10^{-1}$ m the curtain becomes almost opaque for this wave.

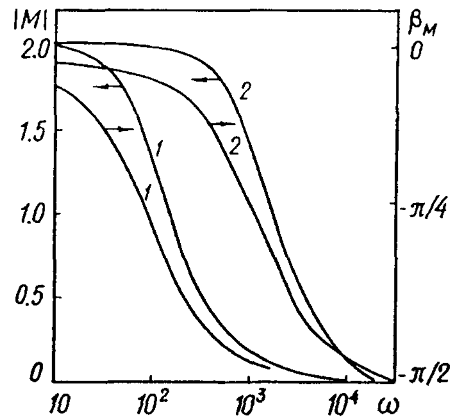
In the case where there is a solid wall in the part to the right ($x > 0$) of the reflecting surface, on the basis of the relations given above we obtain

$$N = \left(\frac{C_p^2}{i\omega l_0 C_{\text{liq}}} + 1\right)\left(\frac{C_p^2}{i\omega l_0 C_{\text{liq}}} - 1\right)^{-1}, \quad M = 1 + N. \tag{17}$$

We note that the coefficient M in this case expresses the ratio between the amplitude of the pressure on the solid wall and the amplitude of the incident wave. Here, also neglecting the effects of radial inertia and viscosity, we can simplify expression (17) to

$$N = \operatorname{ctanh}(i\varphi/2), \quad \tan\varphi = \frac{2S}{S^2 + 1}. \tag{18}$$

In this case, with a change in frequency from 0 to $+\infty$ the hodograph of the coefficient of reflection N in the complex plane is a semicircle in the lower half-plane of radius 1 with center at the origin of coordinates (Fig. 1b). Moreover, for low-frequency waves ($\omega \ll \omega_*$) $N \approx 1$ and, consequently, for reflected perturbations the picture is similar to that in reflection from a rigid wall. For high-frequency perturbations ($\omega \gg \omega_*$) we have $N \approx -1$, $M \approx 0$, and for the reflected wave the curtain is equivalent to a free surface, whereas the solid wall "feels" the incident wave weakly.



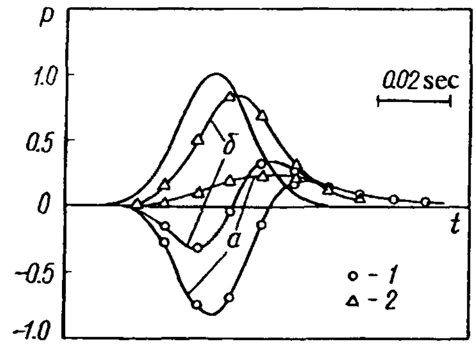


Fig. 3. Dependence of the modulus and argument of the coefficient M for a wall ($|M|$, $\beta_M = \arg M$) on the frequency ω (sec^{-1}) for different volumetric contents of gas in the curtain.

Fig. 4. Evolution of a pressure pulse in interaction with the curtain in the liquid for different volumetric contents of gas in the curtain. p, arb. units; t, sec.

Figure 3 presents dependences of the modulus and argument of the coefficient M on frequency that were calculated from expression (17) (curves 1 and 2 correspond to $\alpha_{g0} = 10^{-1}$ and 10^{-2}, $l_0 = 10^{-1}$ m). It is seen that with increase in the volumetric content of gas in the curtain the modulus of M decreases, and therefore the effect of the harmonic wave on the solid wall decreases.

3. Evolution of Pulse Perturbations. On the basis of the expressions obtained for the coefficients of reflection and transmission we will consider the dynamics of a wave of finite duration in passage through a curtain of bubbles.

Suppose a pressure pulse that has a bell shape and is described by the expression $p^{(0)}(0, t) = \exp(-((t - t_m)/(t_*/2))^2)$ is incident from the left on the reflecting surface ($x < 0$). Here t_* and t_m determine the characteristic duration of the pulse and the moment of time at which the maximum of the amplitude of the primary pulse occurs. We can introduce the parameter λ_* ($\lambda_* = C_{\text{liq}} t_*$), which characterizes the extension in space of the pulse. Using a Fourier transformation, we write the reflected and transmitted signals in the form

$$p^{(r)}(0, t) = \frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} p^{(0)}(0, \tau)\, N(\omega) \exp(i\omega(t - \tau))\, d\omega d\tau \,, \tag{19}$$

$$p^{(t)}(0, t) = \frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} p^{(0)}(0, \tau)\, M(\omega) \exp(i\omega(t - \tau))\, d\omega d\tau \,. \tag{20}$$

Here $p^{(0)}(0, t)$ is the oscillogram of pressure for the incident wave. Results of a numerical realization that were obtained for the evolution of the pressure pulse interacting with a curtain for different volumetric contents of bubbles using the method of rapid Fourier transformation are presented in Fig. 4. Curves a and b are for $\alpha_{g0} = 10^{-1}$ and 10^{-2}, the solid curve is the initial signal, 1 is the reflected pulse, and 2 is the transmitted pulse. It is seen that attenuation of the amplitude of the transmitted pulse occurs much more intensely in the case of a higher volumetric content of gas ($\alpha_{g0} = 10^{-1}$) than in the case of a lower volumetric gas content ($\alpha_{g0} = 10^{-2}$). When $\alpha_{g0} = 10^{-1}$, the transmitted signal is almost completely screened. In this case the reflected-pulse amplitude that corresponds to the high volumetric content of gas ($\alpha_{g0} = 10^{-1}$) exceeds substabtially the amplitude for the curtain with a low volumetric content ($\alpha_{g0} = 10^{-2}$).

Special features of the effect of a pulse signal on a solid wall in relation to its duration for $\alpha_{g0} = 10^{-1}$, $l_0 = 0.1$ m are shown in Fig. 5. It is seen that for a long pulse ($t_* = 0.3$ sec, $\lambda_* = 450$ m) the wall experiences a pulse pressure with an amplitude exceeding the amplitude of the initial signal by about a factor of 1.5 (Fig. 5a), whereas for a short pulse ($t_* = 3 \cdot 10^{-3}$ sec, $\lambda_* = 4.5$ m) the effect on the wall is insignificant in comparison with the initial

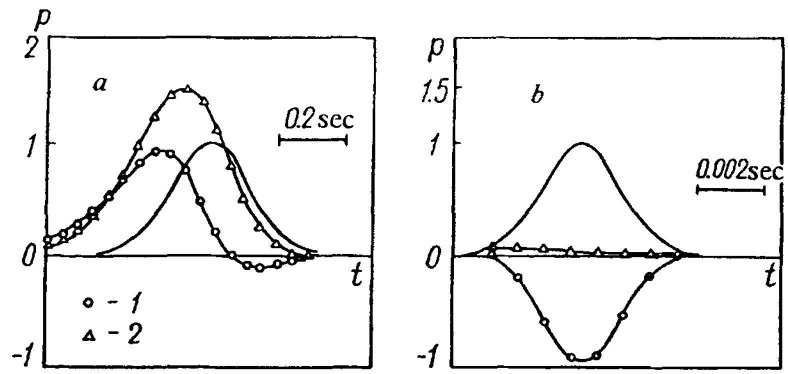


Fig. 5. Effect of a pulse signal on a solid wall in relation to the duration of the pulse: a) $t_* = 0.3$ sec; b) $t_* = 3 \cdot 10^{-3}$ sec; 1, 2) the same as in Fig. 4.

pulse (Fig. 5b). We note that in the absence of a curtain of bubbles the amplitude of the signal on the wall is always twice the amplitude of the incident wave. Consequently, in relation to the duration of the pulse it is possible to select a curtain with corresponding parameters to moderate substantially the effect of the wave on the wall.

NOTATION

ρ^0_{liq}, density of the liquid; α_g, volumetric content; l, thickness of the curtain of bubbles; m, mass of the bubble liquid in a cylindrical volume of unit cross section; n, number of bubbles per unit volume; N, number of bubbles in a cylindrical volume of the curtain of unit cross section; a, radius of the bubbles; ν, kinematic viscosity of the liquid; γ, polytropic exponent; p_{liq}, v, pressure and velocity of the liquid; C_{liq}, speed of sound in the liquid; C_e, equilibrium speed of sound in the liquid with the bubbles; $p^{(0)}$, $p^{(r)}$, $p^{(t)}$ and $v^{(0)}$, $v^{(r)}$, $v^{(t)}$, perturbations of pressure and velocity in the incident, reflected, and transmitted waves; A_p, A_v, amplitudes of perturbations of pressure and velocity; ω, frequency of perturbations; ω_R, resonance frequency of the oscillations of the bubbles; ω_ν, ω_*, characteristic frequencies; k, wave number; x, coordinate; t, time; i, imaginary unit; N, M, coefficients of reflection and transmission; φ, angle in the complex plane; β_N, argument of the reflection coefficient; β_M, argument of the transmission coefficient; t_*, duration of the pulse; λ_*, extension of the pulse in space; τ, integration variable. Subscripts: 0, unperturbed state; N, M, corresponding to parameters of the coefficients of reflection and transmission; R, resonance value; e, equilibrium state; liq, g, parameters of the liquid and the gas; p, v, corresponding to pressure and velocity. Superscripts: (0), (r), (t), corresponding to the original, reflected, and transmitted waves.

REFERENCES

1. G. K. Batchelor, Mekhanika (Periodic Collection of Foreign Papers), 109, No. 3, 67-84 (1968).
2. V. K. Kedrinskii, Prikl. Mekh. Tekh. Fiz., No. 4, 29-34 (1968).
3. V. E. Nakoryakov, B. G. Pokusaev, and I. R. Shreiber, Propagation of Waves in Gas- and Vapor-Liquid Media [in Russian], Novosibirsk (1983).
4. R. I. Nigmatulin, Dynamics of Multiphase Media [in Russian], Pt. 2, Moscow (1987).
5. A. A. Gabaidullin, A. V. Kutrunov, O. Sh. Rustyumova, and T. K. Yakovleva, in: Results of Investigations, Inst. of the Mechanics of Multiphase Systems [in Russian], Tyumen, Issue 5 (1994), pp. 23-31.